Control and Inverse Problem

Stability and control of penetration rate and pressure in rotary drilling system

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Monastir, May 9-11, 2022

Outline

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1 Control system

Definition

$$\dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}, f(0,0) = 0, \quad (1)$$

where

- x : is the state,
- u : is the control law

1. Researchers were interested in stabilizing the system (1) in different ways : exponentially, asymptotic, uniformly asymptotic, partial, in finite time, etc.
2. To achieve these results, the methods involve the following techniques : Lyapunov, Backstepping, LaSalle, etc.

1/2 Examples of control system

Unicycle

Dirigeable

Satellite

1/3 ODEs stability

- M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design,*

  Wiley-Interscience, 1995.
- Hassan K. Khalil, *Nonlinear Systems.*

  Prentice-Hall, third edition, 2002.
- H. J. Marquez, Nonlinear Control, *Systems Analysis and Design,*

  Wiley, 2003.
- S.P. Bhat, D.S. Bernstein, *Finite-time stability of continuous autonomous systems,*

  SIAM J. Control Optim. 38, n.3, 751-766, 2000.

Drilling mechanical system

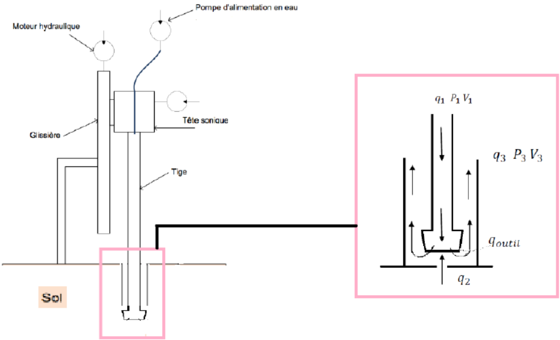

Figure – Drilling mechanical system

Outline

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1. Introduction
2. **Modelisation and Stability of the dynamic pressure**
6. Conclusions and Perspectives

Consider the following equation systems

$$
\begin{aligned}
(2) \quad (V_0 + r^2 \pi y(t)) \dot{P}(t,L) &= \beta \Big[q_1(t,L) + q_2(t,L) - q_3(t,L) - s(t) v(t) \Big] \\
(3) \quad \dot{y}(t) &= v(t) \\
(4) \quad \dot{v}(t) &= U(t)
\end{aligned}
$$

where $y(t) \in [0, L]$ the spatial coordinate along the trajectories of the flow, $V_0 + r^2 \pi y(t)$ is the volume of the crown, β is the compressibility modulus of the mud, $s(t) = r\Phi(t)$, Φ is the angle of the crown, V_0 the volume, v the speed of the slide, q_1 is the pump flow, q_2 is the flow which describes the amount of flow through the surface, s is the surface of the crown, q_3 is the flow out of the crown, P the pressure, and $U(t)$ is the control law.

Using the following variable change

$$z(t) = \frac{1}{V_0 + r^2 \pi y(t)}$$

and let

$$f(t) = \beta \Big[q_1(t, L) + q_2(t, L) - q_3(t, L) \Big]$$

then the equation systems (2)-(4) is transformed in this form

(5) $$\dot{P}(t, L) = f(t) z(t) - \beta s(t) z(t) v(t)$$

(6) $$\dot{z}(t) = -r^2 \pi v(t) z^2(t)$$

(7) $$\dot{v}(t) = U(t).$$

Theorem 1

Let consider the system (5)-(7). We assume that $\frac{\beta P s}{r^2\pi}$ *is never to* $z - z^2$ *and* $q_1(t) + q_2(t) \geq q_3(t)$. *Then the feedback control law*

$$U(t) = -\frac{r^2\pi}{V_0} z^2(t) + \phi(P,z) - v(t) + \dot{\phi}(P,z) + \beta s(t) z(t) P(t) + r^2\pi z^3(t)$$

asymptotically stabilize the system (5)-(7) at the equilibrium $(P,z,v) = (0, \frac{1}{V_0}, 0)$ *with the following Lyapunov function*

$$V_1(P,z,v) = \frac{1}{2}\Big(P^2(t) + \Big(z(t) - \frac{1}{V_0}\Big)^2 + \Big(v(t) - \phi(P,z)\Big)^2\Big),$$

where

$$\phi(P,z) = \frac{P^2(t) + (z(t) - \frac{1}{V_0})^2 + P(t) f(t) z(t)}{\beta P(t) s(t) z(t) + r^2\pi z^3(t) - r^2\pi z^2(t)}$$

in which $\phi(0, \frac{1}{V_0}) = 0$.

Sketch of proof

- Let consider the system

$$\begin{aligned}\dot{P}(t) &= f(t)z(t) - \beta s(t)z(t)v(t)\\ \dot{z} &= -r^2\pi v(t)z^2(t)\end{aligned}$$

and introduce a feedback control law which satisfy

$$v(t) = \phi(P,z) = \frac{P^2(t) + (z(t) - \frac{1}{V_0})^2 + P(t)f(t)z(t)}{\beta P(t)s(t)z(t) + r^2\pi z^3(t) - r^2\pi z^2(t)}$$

- First, we use the following Lyapunov function

$$V(P,z) = \frac{1}{2}(P^2 + (z - \frac{1}{V_0})^2).$$

- Second, we use the following backstepping transformation,

$$\zeta(t) = v(t) - \phi(P,z)$$

Global model of the dynamic pressure

In this section we present the global model of the dynamic pressure

$$
\begin{aligned}
M\dot{q}_{outil}(t) &= P_1(t) - P_3(t) - F(q_{outil}) + (\rho_1 - \rho_3) g h_{fond} y \\
\frac{V_1}{\beta_1}\dot{P}_1(t) &= u_1(t) - q_{outil}(t) \\
(V_0 + r^2 \pi y(t))\dot{P}_3(t) &= \beta\Big[q_{outil}(t) + q_2(t) - q_3(t) - s(t)v(t)\Big] \\
\dot{y}(t) &= v(t) \\
\dot{v}(t) &= u_2(t).
\end{aligned}
$$

such that u_1 is the first control law and u_2 is the second control law. The goal of the both control law is to controller the pressure , the speed and the position of the glisiere.

We linearize the function F around zero and we use the following variable change $z(t) = \dfrac{1}{V_0 + r^2 \pi y(t)}$. Then we obtain the following equation systems

Global model of the dynamic pressure

$$
\begin{aligned}
&(8) \quad & \dot{q}_{outil}(t) &= c_1 P_1(t) - c_1 P_3(t) + h(z) \\
&(9) \quad & \dot{P}_1(t) &= c u_1(t) - c q_{outil}(t) \\
&(10) \quad & \dot{P}_3(t) &= f(t) z(t) - \beta_3 z(t) s(t) v(t) \\
&(11) \quad & \dot{z}(t) &= -r^2 \pi z^2(t) v(t) \\
&(12) \quad & \dot{v}(t) &= u_2(t)
\end{aligned}
$$

where

$c = \frac{\beta_1}{V_1}$, $\frac{1}{M+F'(0)} = c_1$, $h(z) = \frac{c_1}{r^2\pi}(\rho_1 - \rho_3) g h_{fond}(\frac{1}{z(t)} - V_0)$,

$f(t) = \beta_3 \left[q_{outil}(t) + q_2(t) - q_3(t) \right]$

Theorem 2

Let consider the system (8)-(12). We use the assumption of theorem 1, then the both feedback control law

$$u_1(t) = -\frac{\eta}{c} + \frac{c - c_1}{c} q_{outil}(t) + \frac{1}{c}\dot{\psi}(q_{outil}, P_3, z)$$

and

$$u_2(t) = -\frac{r^2\pi}{V_0} z^2(t) + \phi(P_3, z) - v(t) + \dot{\phi}(P_3, z) + \beta_3 s(t) z(t) P_3(t) + r^2\pi z^3(t)$$

asymptotically stabilize the system equations (8)-(12) with the following Lyapunov function

$$V_2 = \frac{1}{2}\Big(q_{outil}^2 + \big(P_1 - \psi(q_{outil}, P_3, z)\big)^2\Big) + \frac{1}{2}\Big(P_3^2(t) + \big(z(t) - \frac{1}{V_0}\big)^2 + \big(v(t) - \phi(P_3, z)\big)^2\Big),$$

where

$$\psi(q_{outil}, P_3, z) = P_3 - \frac{q_{outil}}{c_1} - \frac{h(z)}{c_1}, \quad \phi(P_3, z) = \frac{P_3^2(t) + (z(t) - \frac{1}{V_0})^2 + P_3(t) f(t) z(t)}{\beta P_3(t) s(t) z(t) + r^2\pi z^3(t) - r^2\pi z^2(t)}$$

in which $\phi(0, \frac{1}{V_0}) = 0, \quad \psi(0, 0, \frac{1}{V_0}) = 0.$

Sketch of proof

- First, consider the following equation

$$\dot{q}_{outil}(t) \quad = \quad c_1 P_1(t) - c_1 P_3(t) + h(z)$$

and introduce a feedback control law which satisfy

$$\psi(q_{outil}, P_3, z) = P_1 = P_3 - \frac{q_{outil}}{c_1} - \frac{h(z)}{c_1}$$

in which $\psi(0, 0, \frac{1}{V_0}) = 0$,

- Second, let consider the following backstepping transformation

$$\eta = P_1 - \psi(q_{outil}, P_3, z).$$

- Finally, we say that the system (8)-(12) is asymptotically stable at the equilibrium (see Theorem 1).

(13) $$\dot{q}_{outil}(t) = c_1 P_1(t) - c_1 P_3(t) + h(z)$$

(14) $$\dot{P}_1(t) = c u_1(t) - c q_{outil}(t)$$

(15) $$\dot{P}_3(t) = g(t) z(t) + \beta_3 u_3(t) z(t) - \beta_3 z(t) s(t) v(t)$$

(16) $$\dot{z}(t) = -r^2 \pi z^2(t) v(t)$$

(17) $$\dot{v}(t) = u_2(t).$$

where $g(t) = \beta_3 \left[q_{outil}(t) - q_3(t) \right]$

Theorem 3

Let consider the system (13)-(17). Then the three feedback control law

$$\begin{aligned} u_1(t) &= \frac{-\eta}{c} + \frac{c - c_1}{c} q_{outil}(t) + \frac{1}{c}\dot{\psi}(q_{outil}, P_3, z) \\ u_2(t) &= r^2 \pi z^2(t)(z - \frac{1}{V_0}) + \dot{\Phi}_1(z) - v + \Phi_1(z) \\ u_3(t) &= s(t)v(t) + \frac{P_3}{\beta_3 z(t)} - \frac{1}{\beta_3} g(t) \end{aligned}$$

asymptotically stabilize the system (13)-(17) at the equilibrium
$(q_{outil}, P_1, P_3, z, v) = (0, 0, 0, \frac{1}{V_0}, 0)$
with the following Lyapunov function

$$V_7 = \frac{1}{2}\left(q_{outil}^2 + (P_1 - \psi(q_{outil}, P_3, z))^2 + P_3^2 + (v - \Phi_1(z))^2 + (z - \frac{1}{V_0})^2\right)$$

where

$$\Phi_1(z) = \frac{z - \frac{1}{V_0}}{r^2 \pi z^2}, \quad \psi(q_{outil}, P_3, z) = P_3 - \frac{q_{outil}}{c_1} - \frac{h(z)}{c_1}$$

Conclusions & Perspectives

- NL control of the Torsional vibration
- Identification and Control for the different model's parameters
- Improve the Control performance
- Stability of the speed penetration with table dynamics
- Take into account the mud dynamic
- Observers Design for the control problem

Thank you for attention