SINGULAR LIMIT SOLUTIONS FOR A 2-DIMENSIONAL SEMILINEAR ELLIPTIC SYSTEM OF LIOUVILLE TYPE IN SOME GENERAL CASE ADDING SINGULAR SOURCES

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Abstract. Let Ω be a regular bounded open domain in $\mathbb{R}^2$. We consider the following elliptic system

$$\left\{\begin{array}{lcll} -\Delta v_1 & = & \rho^2 e^{\gamma v_1+(1-\gamma)v_2} - \frac{4\pi}{\gamma}\,\alpha_1\delta_{p_1} - 4\pi\,\alpha_2\delta_{p_2} & \text{in} \quad \Omega \\ -\Delta v_2 & = & \rho^2 e^{\xi v_2+(1-\xi)v_1} - \frac{4\pi}{\xi}\,\alpha_3\delta_{p_3} - 4\pi\,\alpha_2\delta_{p_2} & \text{in} \quad \Omega \\ v_1 & = & v_2 \; = \; 0 & \text{on} \quad \partial\Omega. \end{array}\right.$$

Here γ, ξ and ρ are constants and for $i \in \{1,2,3\}$, α_i are positive non-integer numbers. We denote by $\Lambda = \{p_1, p_2, p_3\} \subset \Omega$, the set of singular sources. We assume that $\gamma, \xi \in (0,1)$ such that $\gamma + \xi > 1$. So in all the following, we have

$$\frac{1-\xi}{\gamma} \text{ and } \frac{1-\gamma}{\xi} \in (0,1).$$

The purpose of this talk is to prove the existence of a coupled solution (v_1, v_2) for the previous problem. We are interested to the study of the existence of this solution with singular limits as the parameter ρ tends to 0.