SINGULAR LIMIT SOLUTIONS FOR A
2-DIMENSIONAL SEMILINEAR ELLIPTIC SYSTEM OF
LIouvillE TYPE IN SOME GENERAL CASE
ADDING SINGULAR SOURCES

SAMI BARAKET

ABSTRACT. Let \( \Omega \) be a regular bounded open domain in \( \mathbb{R}^2 \). We consider the following elliptic system
\[
\begin{aligned}
-\Delta v_1 &= \rho^2 e^{\gamma v_1 + (1-\gamma)v_2} - \frac{4\pi}{\gamma} \alpha_1 \delta_{p_1} - 4\pi \alpha_2 \delta_{p_2} \quad \text{in} \quad \Omega \\
-\Delta v_2 &= \rho^2 e^{\xi v_2 + (1-\xi)v_1} - \frac{4\pi}{\xi} \alpha_3 \delta_{p_3} - 4\pi \alpha_2 \delta_{p_2} \quad \text{in} \quad \Omega \\
v_1 &= v_2 = 0 \quad \text{on} \quad \partial \Omega.
\end{aligned}
\]
Here \( \gamma, \xi \) and \( \rho \) are constants and for \( i \in \{1, 2, 3\} \), \( \alpha_i \) are positive non-integer numbers. We denote by \( \Lambda = \{p_1, p_2, p_3\} \subset \Omega \), the set of singular sources. We assume that \( \gamma, \xi \in (0, 1) \) such that \( \gamma + \xi > 1 \). So in all the following, we have
\[
\frac{1-\xi}{\gamma} \quad \text{and} \quad \frac{1-\gamma}{\xi} \in (0, 1).
\]
The purpose of this talk is to prove the existence of a coupled solution \((v_1, v_2)\) for the previous problem. We are interested to the study of the existence of this solution with singular limits as the parameter \( \rho \) tends to 0.